# palette of problems

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1. A rectangular prism with edges that are natural numbers has a total volume of 100 cubic centimeters. The longest side of the prism is twice as long as the next longest side, and the area of the base is 10 square centimeters. What are the dimensions of the prism?

2. The diagram illustrates the "two monks tangram paradox." In the two pictures, figures of the same color are identical. Which of the "monks" has a larger total area?
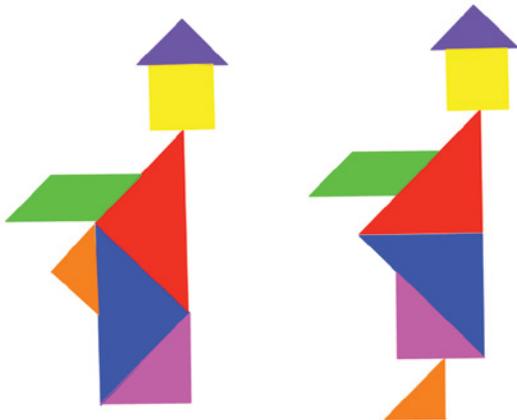
3. Jacob and Patty are starting a lawn mowing business for the summer. On average, Jacob can mow 4 lawns in 6 hours, and Patty can mow 5 lawns in 8 hours. Assuming that they both spend the same amount of time working, how long will it take Jacob and Patty to mow 62 lawns?

4. Suppose that, when divided by 4, the number a has a remainder of 2 and the number b has a remainder of 3. What will be the remainder of the product ab when divided by 4?

5. Your sock drawer contains 6 socks, all of which are either blue or red. Without looking, you take 2 socks out of the drawer. If your probability of drawing a pair of blue socks is 2/3, how many blue socks are in the drawer?

6. The taxicab distance between two points in the plane with coordinates (a, b) and (c, d), where a, b, c, and d are integers, is defined as $|a - c| + |b - d|$ (see the March "Palette of Problems" for a more complete explanation). A circle can be defined as the set of points some fixed distance (called the radius) from a center. Using taxicab distance, graph a "circle" of radius 3 centered at the origin.

7. If a can be chosen from the set {2, 3, 6, 9}, b can be chosen from the set {4, 5, 6, 7}, and c can be chosen from the set {1, 2, 4, 5}, what values of a, b, and c will produce the largest solution for the equation $ax + b = c$? What values of a, b, and c will produce the smallest solution?

8. How long would it take to count to 1 billion? What assumptions must you make before proceeding? Do you think it would take 1 hour? One day? Perhaps 1 week? Maybe longer?

9. A bug is crawling across my graph paper in a straight line. When I first notice it, the bug is at the location (–3, –4). Five seconds later, it is at the location (1, 1). Assuming that the bug crawls at a constant rate, what are the coordinates of the point where the bug is located after 1 minute?

10. An imaginary island is called the Island of Knights and Knaves. On this island, there are people called knights, who always tell the truth, and people called knaves, who always lie. The two types are indistinguishable by sight. On a vacation to this island, you meet two inhabitants: Bill and Bob. Bob says, "We are both the same kind," but Bill says, "We are both different kinds." Who is the knight, and who is the knave?

11. The line L has equation $2y = 100 - 5x$. Find the number of points on line L whose coordinates are both positive whole numbers.

12. A pile of nickels and quarters is on a table. The total amount of money in the pile is more than 0 and less than 1 dollar. If all the coins (both the nickels and quarters) were turned into dimes, the amount of money in the pile would be unchanged. How many coins could there be in the pile?

13. Nine players line up single file to shoot free throws. Exactly 4 of the players are seniors. In how many ways can the line be formed if the first 2 and the last 2 people in the line are seniors?

14. A 1/2 mile long train is going at 60 miles per hour and is just entering a tunnel that is 1 mile in length. How many minutes will it take the entire train to go completely through the tunnel?

15. A 5 inch square is inscribed in a circle. What is the area between one side of the square and an arc of the circle?
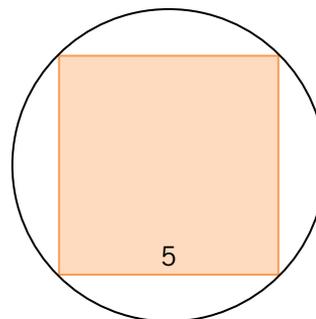
16. Two sides of a triangle have lengths 2 and 3 and the third side has length L. What could L be if the triangle has only acute angles?

(Answers on page 574)